

# Crystallized Rates Region of the Interference Channel via Correlated Equilibrium with Interference as Noise

Mohamad Charafeddine, Zhu Han\*, Arogyaswami Paulraj and John Cioffi,  
Electrical Engineering Department, Stanford University, CA, USA

\*Electrical and Computer Engineering Department, University of Houston, Houston, USA

**Abstract**—Treating the interference as noise in the  $n$ -user interference channel, the paper describes a novel approach to the rates region, composed by the time-sharing convex hull of  $2^n - 1$  corner points achieved through On/Off binary power control. The resulting rates region is denoted *crystallized rates region*. By treating the interference as noise, the  $n$ -user rates region frontiers has been found in the literature to be the convex hull of  $n$  hyper-surfaces. The rates region bounded by these hyper-surfaces is not necessarily convex, and thereby a convex hull operation is imposed through the strategy of time-sharing. This paper simplifies this rates region in the  $n$ -dimensional space by having only an On/Off binary power control. This consequently leads to  $2^n - 1$  corner points situated within the rates region. A time-sharing convex hull is imposed onto those corner points, forming the crystallized rates region. The paper focuses on game theoretic concepts to achieve that crystallized convex hull via correlated equilibrium. In game theory, the correlated equilibrium set is convex, and it consists of the time-sharing mixed strategies of the Nash equilibriums. In addition, the paper considers a mechanism design approach to carefully design a utility function, particularly the Vickrey-Clarke-Groves auction utility, where the solution point is situated on the correlated equilibrium set. Finally, the paper proposes a self learning algorithm, namely the regret-matching algorithm, that converges to the solution point on the correlated equilibrium set in a distributed fashion.

## I. INTRODUCTION

Wireless systems are becoming increasingly interference limited rather than noise limited, attributed to the fact that the cells are decreasing in size and the number of users within a cell is increasing. Mitigating the impact of interference between transmit-receive pairs is of great importance in order to achieve higher data rates. Describing the complete capacity region of the interference channel remains an open problem in information theory [1]–[5]. For very strong interference, successive cancellation schemes have to be applied, while in the weak interference regime, treating the interference as additive noise is optimal to within one bit [6]–[9]. Treating the interference as noise, the  $n$ -user achievable rates region has been found in [10] to be the convex hull of  $n$  hyper-surfaces. The rates region bounded by these hyper-surfaces is not necessarily convex, and hence a convex hull operation is imposed through the strategy of time-sharing.

This paper adopts a novel approach into simplifying this rates region in the  $n$ -dimensional space by having only On/Off binary power control. Limiting each of the  $n$  transmitters to a transmit power of either 0 or  $P_{\max}$ , this consequently leads to  $2^n - 1$  corner points within the rates region. And by forming a convex hull through time-sharing between those corner points, it thereby leads to what we denote a crystallized rates region.

Utility maximization using game-theoretic techniques has recently received significant attention [11]–[15]. Most of the existing game theoretic works are based on the concept of Nash

equilibrium [16]. However, the Nash equilibrium investigates the individual payoff and might not be system efficient, i.e. the performance of the game outcome could still be improved. In 2005, Nobel Prize was awarded to Robert J. Aumann for his contribution of proposing the concept of correlated equilibrium [17]. Unlike Nash equilibrium in which each user only considers its own strategy, correlated equilibrium achieves better performance by allowing each user to consider the joint distribution of the users' actions. In other words, each user needs to consider the others' behaviors to see if there are mutual benefits to explore [18]–[20]. Likewise, mechanism design (including auction theory [21]) is a subfield of game theory that studies how to design the game rule in order to achieve good overall system performance [22], [23]. Mechanism design has drawn recently a great attention in the research community, especially after another Nobel Prize in 2007.

The paper presents three contributions with the following structure:

- 1) Section II introduces the concept of crystallized rates region with On/Off power control.
- 2) Section III applies the game theoretic concept of correlated equilibrium (CE) to the rates region problem. The CE exhibits the property of forming a convex set around the  $2^n - 1$  corner points, hence fitting suitably in the crystallized rates region formulation.
- 3) Using mechanism design, Section IV presents an example in applying these two concepts for the 2-user channel and formulates the Vickrey-Clarke-Groves auction utility. To find the solution point distributively, the regret matching learning algorithm is employed by virtue of its property of converging to the correlated equilibrium set.

Section V demonstrates the ideas through simulation, and Section VI draws the conclusions.

## II. CRYSTALLIZED RATES REGION

### A. System Model for 2-user Interference Channel

A 2-user interference channel is illustrated in Fig. 1. User  $i$  transmits its signal  $X_i$  to the receiver  $Y_i$ . The receiver front end has additive thermal noise  $n_i$  of variance  $\sigma_n^2$ . There is no cooperation at the transmit, nor at the receive side. The channel is flat fading. For brevity  $a$ ,  $b$ ,  $c$ , and  $d$  represent the channel power gain *normalized* by the noise variance. Explicitly,  $a = |g_{1,1}|^2/\sigma_n^2$ ,  $b = |g_{2,1}|^2/\sigma_n^2$ ,  $c = |g_{2,2}|^2/\sigma_n^2$ , and  $d = |g_{1,2}|^2/\sigma_n^2$ , where  $g_{i,j}$  is the channel gain from the  $i^{th}$  transmitter to the  $j^{th}$  receiver. User  $i$  transmits with power  $P_i$ , and it has a maximum power constraint of  $P_{\max}$ .

In an effort to keep the complexity of the receivers fairly simple, the interference is treated as noise. Such case is encountered in sensor networks and in cellular communication where

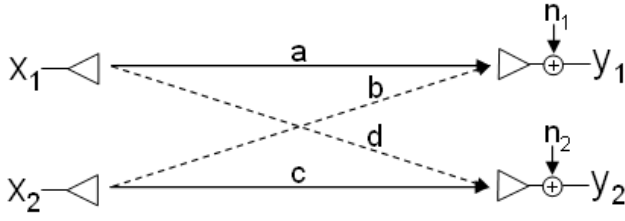


Fig. 1: 2-user interference channel

it is desired to have low power-consuming and correspondingly low complexity receivers. Therefore, with the power vector  $\mathbf{P} = [P_1, P_2]^T$ , treating the interference as noise the achievable rates for the 2-user interference channel are:

$$\begin{aligned} R_1(\mathbf{P}) &= \log_2 \left( 1 + \frac{aP_1}{1 + bP_2} \right); \\ R_2(\mathbf{P}) &= \log_2 \left( 1 + \frac{cP_2}{1 + dP_1} \right). \end{aligned} \quad (1)$$

#### B. The Achievable Rates Region Treating Interference as Noise

In [10] the achievable rates region for the general  $n$ -user channel, treating the interference as noise, is found to be the convex hull of the union of  $n$  hyper-surfaces. Each hyper-surface is characterized by holding one of the transmitter constant at a full power while the other transmitters sweep all their power range, hence forming a hyper-surface as a result. There are  $n$  transmitters, resulting in  $n$  hyper-surfaces, onto which the convex hull operation is performed.

The convexity or concavity behavior of these hyper-surface frontiers is complex. A rates region set is convex whenever it entirely encloses a straight line formed by connecting any two points within the rates region. As a result when the rates region is *convex*, its outerbound hyper-surface frontiers are *concave*, and vice versa.

For the 2-user channel, see Fig. 2, the hyper-surfaces are the two frontiers:  $\Phi_{AB} = \Phi(:, P_{\max})$ , characterized by holding  $P_2 = P_{\max}$  and  $P_1$  sweeps all its power range from 0 to  $P_{\max}$ , and  $\Phi_{BC} = \Phi(P_{\max}, :)$  characterized by holding  $P_1 = P_{\max}$  and  $P_2$  sweeps all its power range from 0 to  $P_{\max}$ . These frontiers are referred to as *potential lines* given that each is characterized by holding one the transmit power arguments at a constant value, in this case  $P_{\max}$ , while the other power argument spans the whole power range.

These potential lines are concave in noise-limited regimes (thus enclosing a convex rates region) as in Fig. 2-a, and they shift towards convexity as the interference increases, as in Fig. 2-d. In cases with moderate interference levels, they can exhibit non-stationary inflection point, as at point D in Fig.2-b.

#### C. Crystallized Rates Region

The crystallized rates region approach approximates the achievable rates region formed by the potential lines  $\Phi_{AB}$  and  $\Phi_{BC}$  into the convex time-sharing hull of the straight lines connecting points A, B, and C. Denoting  $\Phi(P_1, P_2)$  the point in the rates region achieved when user 1 transmits at  $P_1$  and user 2 transmits at  $P_2$  in Eq. (1): point A is  $\Phi(0, P_{\max})$  where only user 2 transmits at full power and user 1 is silent, point B is  $\Phi(P_{\max}, P_{\max})$  where both users transmit simultaneously at full power, and point C is  $\Phi(P_{\max}, 0)$  where user 1 transmits at full power and user 2 is silent. Hence, we refer by binary power

control such mechanism of operation in producing points A, B, and C. We denote such points that are formed by binary power control as the *corner* points of the rates region. In the 2-user interference channel, there exist 3 corner points, similarly in the  $n$ -user case there exist  $2^n - 1$  corner points.

Therefore, this paper simplifies the analysis of the rates region in the  $n$ -dimensional space to just focus on finding the convex time-sharing hull onto the  $2^n - 1$  corner points, forming the crystallized rates region. In the 2-user dimension, these are straight time-sharing lines connecting two points; in the 3-user dimension, these are a set of polygon surfaces each connecting three points, see Fig. 3.

#### D. System Time-sharing Coefficients and Rates Equations

Instead of a power control problem in finding  $P_i$ , the problem becomes finding the appropriate time-sharing coefficients of the  $2^n - 1$  corner points. For the 2-user case, let  $\theta = [\theta_1, \theta_2, \theta_3]^T$ ,  $\sum_i \theta_i = 1$ , denote the *system* time-sharing coefficients vector of the respective corner points  $\Phi(P_{\max}, 0)$  (user 1 transmitting only with a time-sharing coefficient  $\theta_1$ ),  $\Phi(0, P_{\max})$  (user 2 transmitting only with a time-sharing coefficient  $\theta_2$ ), and  $\Phi(P_{\max}, P_{\max})$  (both users transmitting with a time-sharing coefficient  $\theta_3$ ). The reason  $\theta$  is labeled a *system* time-sharing coefficients vector is to emphasize the combinatorial element in constructing the corner points, where the cardinality of  $|\theta| = 2^n - 1$ .

Then for 2-user case, in contrast with Eq. (1), the new crystallized rates equations for  $R_1$  and  $R_2$  are:

$$\begin{aligned} R_1(\theta) &= \theta_1 \log_2(1 + aP_{\max}) + \theta_3 \log_2 \left( 1 + \frac{aP_{\max}}{1 + bP_{\max}} \right) \\ R_2(\theta) &= \theta_2 \log_2(1 + cP_{\max}) + \theta_3 \log_2 \left( 1 + \frac{cP_{\max}}{1 + dP_{\max}} \right) \end{aligned} \quad (2)$$

Any solution point on the crystallized frontier would lie somewhere on the time-sharing line connecting two points for the 2-user case; and similarly for the 3-user case, the solution point lies somewhere on a time-sharing plane connecting three points, then by deduction we obtain the following corollary:

*Corollary 1:* The system time-sharing vector  $\theta$ , for any solution point on the  $n$ -user crystallized rates region, has at maximum  $n$  nonzero coefficients out of its  $2^n - 1$  elements.

#### E. Evaluation of Crystallization

Examining the crystallized rates region in more details for the 2-user interference channel, we evaluate the area of the rates region bounded by the potential lines  $\Phi_{AB}$  and  $\Phi_{BC}$  achieved through power control, and the area of the rates region formed by time-sharing points A, B, and C. In effect, we are evaluating how much gain or loss results from completely replacing the traditional power control scheme (see Eq. (1)) with the time-sharing scheme between the corner points (see Eq. (2)). For this purpose we consider the symmetric channel, where  $a = 1$ , and we increase the interference  $b$  to vary the signal to interference ratio  $SIR = a/b$  from 20dB to -20dB. The value of the area bounded by the power control potential lines is plotted in Fig.4 together with the value of the area bounded by the time-sharing scheme through the point B (formed by the time-sharing lines A-B and B-C). In addition,

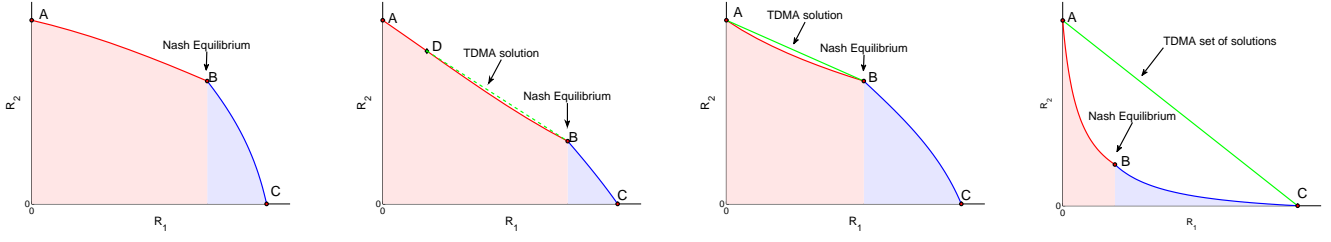


Fig. 2: 2-user rates region: (a) noise-limited, concave frontiers ( $\Phi_{AB}, \Phi_{BC}$ ); (b) frontier ( $\Phi_{AB}$ ) with inflection-point; (c) convex ( $\Phi_{AB}$ ) and concave ( $\Phi_{BC}$ ) frontiers; (d) interference-limited, convex frontiers ( $\Phi_{AB}, \Phi_{BC}$ ).

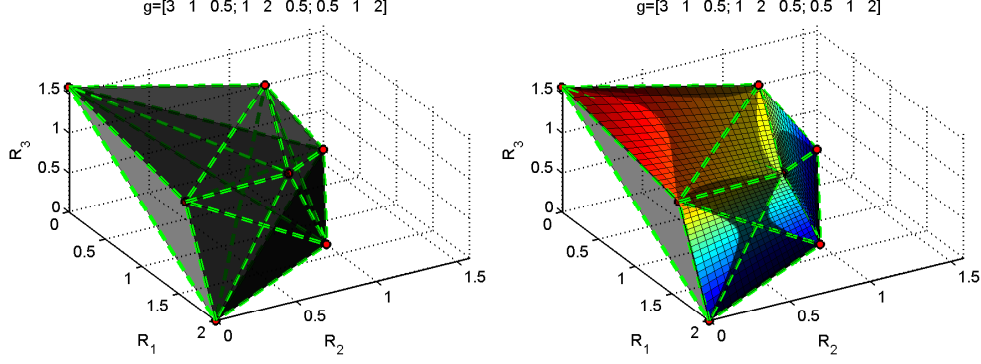


Fig. 3: 3-user crystallized rates region: (a) time-sharing crystallized hull, (b) crystallized hull overlaid on top of the rates region

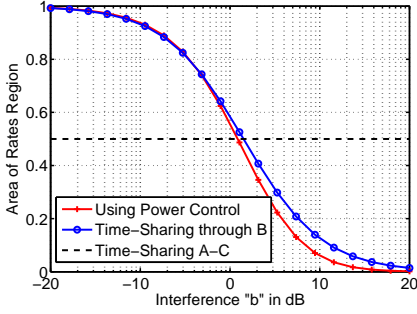


Fig. 4: Area of the rates region achieved through power control or through time-sharing versus interference

for reference, the area confined by the time-sharing line A-C is plotted, which does not depend on the SIR.

For weak interference, or equivalently noise-limited regime, point B is used in constructing the crystallized region. As the interference increases beyond a certain threshold level, time-sharing through point B becomes suboptimal, and time-sharing A-C becomes optimal. The exact switching point from power control to time-sharing has been found in [10]. In Fig. 4, this happens at the intersection of the blue line (with circle markers) and the A-C dotted line. As indicated in Fig. 4, there is no significant loss in the rates region area if time-sharing is used universally instead of traditional power control, in fact in some cases time-sharing offers considerable gain. Specifically, whenever the potential lines exhibit concavity, time-sharing loses to power control; whenever the potential lines exhibit convexity, time-sharing gains over power control. Different values of  $a$  also lead to the same conclusion.

In Fig. 5 the percentage of the rates region gain (or loss) in using the time-sharing scheme (through point B) over

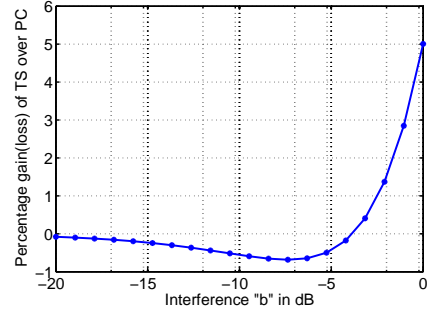


Fig. 5: Gain (or loss) percentage from using time-sharing through point B over power control

the power control scheme is plotted for the same symmetric channel examined in Fig. 4. The loss does not exceed 1%, and the time-sharing strategy is therefore quite attractive. For illustration purposes, note that the x-axis in Fig. 5 was chosen to span the interference range of  $-20\text{dB}$  to  $0\text{dB}$ ; whereas in Fig. 4 the x-axis interference range was from  $-20\text{dB}$  to  $20\text{dB}$ . If we were to plot the x-axis in Fig. 5 up to  $20\text{dB}$  instead of the  $0\text{dB}$ , the percentage gain would have reached on the y-axis up to  $800\%$ . Note that for high interference time-sharing through point B is suboptimal and time-sharing A-C is optimal, so the gain over power control is even larger.

### III. CORRELATED EQUILIBRIUM FOR CRYSTALLIZED INTERFERENCE CHANNEL

The crystallized rates region offers a good alternative to form the rates region of the  $n$ -user interference channel with marginal loss, and sometimes significant gain (especially for interference-limited regimes). Therefore the problem revolves around finding the convex hull over the set of polygons

connecting the  $2^n - 1$  corner points. One technique explored to achieve the convex hull is through the concept of correlated equilibrium in game theory.

#### A. Correlated Equilibrium

Every user  $i$  has a transmit strategy  $\alpha_i$  of either 0 or  $P_{\max}$ .  $U_i$  is the utility of user  $i$ . Nash equilibrium is a well-known concept to analyze the outcome of a game which states that in the equilibrium every user will select a utility-maximizing strategy given the strategies of every other user.

*Definition 1:* Nash equilibrium achieving strategy  $\alpha_i^*$  of user  $i$  is defined as:

$$U_i(\alpha_i^*, \alpha_{-i}) \geq U_i(\alpha_i, \alpha_{-i}), \forall i, \forall \alpha_i \in \Omega_i \quad (3)$$

where  $\alpha_i$  is any possible strategy of user  $i$ ,  $\alpha_{-i}$  is the strategy vector of all other users except user  $i$ , and  $\Omega_i$  is the strategy space  $\{0, P_{\max}\}$ . In other words, given the other users' actions, no user can increase its utility alone by changing its own action.

Next the concept of the correlated equilibrium is studied. It is more general than the Nash equilibrium and it was first proposed in [17]. The idea is that a strategy profile is chosen according to the joint distribution instead of the marginal distribution of users strategies. When converging to the recommended strategy, it is to the users' best interests to conform to this strategy. The distribution is called the correlated equilibrium, which is defined as:

*Definition 2:* A probability distribution  $p$  is a correlated equilibrium of a game, if and only if, for all  $i$ ,  $\alpha_i \in \Omega_i$ , and  $\alpha_{-i} \in \Omega_{-i}$ ,

$$\sum_{\alpha_{-i} \in \Omega_{-i}} p(\alpha_i^*, \alpha_{-i}) [U_i(\alpha_i^*, \alpha_{-i}) - U_i(\alpha_i, \alpha_{-i})] \geq 0, \forall \alpha_i \in \Omega_i. \quad (4)$$

$\Omega_{-i}$  denotes the strategy space of all the users other than user  $i$ . As every user  $j$ ,  $j \neq i$ , has a possible 0 or  $P_{\max}$  strategy choice, then the cardinality of  $\Omega_{-i}$  is  $|\Omega_{-i}| = 2^{(n-1)}$ . Therefore the summation in Eq. (4) have  $2^{n-1}$  summation terms. The summation over  $\alpha_{-i}$  generates the marginal expectation. The inequality (4) means that when the recommendation to user  $i$  is to choose action  $\alpha_i^*$ , then choosing action  $\alpha_i$  instead of  $\alpha_i^*$  cannot result in a higher expected payoff to user  $i$ . It is worth to point out that the probability distribution  $p$  is a joint point mass function (pmf) of the different combinations of the  $n$  users strategies. Therefore,  $p$  is the joint pmf of the resulting  $2^n$  system strategy points. Discounting the trivial system strategy of all the users transmitting at 0, there exist  $2^n - 1$  system strategy points that we wish to find their pmfs.

#### B. CE in the Context of the Crystallized Rates Region

Revisiting Subsection II-D, the  $2^n - 1$  point mass functions that we want to find are the system time-sharing coefficients  $\theta_k$ ,  $k = 1, \dots, 2^n - 1$ . We can index those  $2^n - 1$  pmfs to the corresponding  $\theta_k$  in any bijective one-to-one mapping. Index  $k$  can denote the base-2 representation of the binary users' strategies (starting with user 1's binary action as the least significant bit). For example, let  $\alpha_i^{(1)}$  denotes that user  $i$  transmits with  $\alpha_i = P_{\max}$ , and  $\alpha_i^{(0)}$  denotes that user  $i$  is silent with  $\alpha_i = 0$ . In Subsection II-D,  $\theta_1$  was mapped to user 1 transmitting, equivalently  $\theta_1 = p(\alpha_1^{(1)}, \alpha_2^{(0)}) = p_{\Phi(P_{\max}, 0)}$ ;

where we defined explicitly  $p_{\Phi(P_{\max}, 0)}$  as the point mass function of the point  $\Phi(P_{\max}, 0)$ . And similarly  $\theta_3$  was mapped to both users transmitting,  $\theta_3 = p(\alpha_1^{(1)}, \alpha_2^{(1)}) = p_{\Phi(P_{\max}, P_{\max})}$ . Moreover, by definition,  $\sum_{\alpha} p(\alpha) = \sum_{k=1}^{(2^n-1)} \theta_k = 1$ , and as discussed in Corollary I, the solution point possesses at most  $n$  nonzero pmfs in the joint distribution  $p$ .

The correlated equilibriums set is nonempty, closed and convex in every finite game [17]. In fact, every Nash equilibrium and mixed (i.e. time-sharing) strategy of Nash equilibriums are within the correlated equilibrium set, and the Nash equilibrium correspond to the special case where  $p(\alpha)$  is a product of each individual user's probability for different actions, i.e., the play of the different users is independent [17], [23].

### IV. MECHANISM DESIGN AND LEARNING ALGORITHM

There are two major challenges to implement correlated equilibrium for rate optimization over the interference channel. First, to ensure the system converges to the desired point (such as time-sharing between A-C instead of going through point B in Fig. 2 (d)). As an example, we considered an auction utility function from mechanism design. Second, to achieve the equilibrium, a distributive solution is desirable, where we propose the self-learning regret matching algorithm.

#### A. Mechanism Designed Utility

One important mechanism design is the Vickrey-Clarke-Groves (VCG) auction [21] which imposes cost to resolve the conflicts between users. Using the basic idea of VCG, where we want to maximize  $U_i, \forall i$ , the user utility  $U_i$  is designed to be the rate  $R_i$  minus a payment cost function  $\zeta_i$  as

$$U_i \triangleq R_i - \zeta_i. \quad (5)$$

The payment cost of user  $i$  is expressed as the performance loss of all other users due to the inclusion of user  $i$ , explicitly:

$$\zeta_i(\alpha) = \sum_{j \neq i} R_j(\alpha_{-i}) - \sum_{j \neq i} R_j(\alpha_i). \quad (6)$$

Hence if  $\alpha_i$  is 0 for user  $i$ , it is equivalent to user  $i$  being absent, consequently the cost  $\zeta_i = 0$  whenever  $\alpha_i = 0$ . For the 2-user case, focusing on  $\zeta_1$  when  $\alpha_1 = P_{\max}$ , hence: a) if  $\alpha_2 = 0$ , then  $R_2 = 0$  and  $\zeta_1 = 0$ ; b) if  $\alpha_2 = P_{\max}$ , then:

$$\begin{aligned} \zeta_1(\alpha_1 = P_{\max}, \alpha_2 = P_{\max}) &= R_2(\alpha_1 = 0, \alpha_2 = P_{\max}) - R_2(\alpha_1 = P_{\max}, \alpha_2 = P_{\max}) \\ &= \log_2 \left( 1 + cP_{\max} \right) - \log_2 \left( 1 + \frac{cP_{\max}}{1 + dP_{\max}} \right) \\ &= \log_2 \left( 1 + \frac{cdP_{\max}^2}{1 + cP_{\max} + dP_{\max}} \right). \end{aligned}$$

$\zeta_2$  follows by symmetry. As a result, the VCG utilities for the 2-user channel are summarized in Table I, where

$$\begin{aligned} U'_1 &= \log_2 \left( 1 + \frac{aP_{\max}}{1 + bP_{\max}} \right) - \log_2 \left( 1 + \frac{cdP_{\max}^2}{1 + cP_{\max} + dP_{\max}} \right) \\ U'_2 &= \log_2 \left( 1 + \frac{cP_{\max}}{1 + dP_{\max}} \right) - \log_2 \left( 1 + \frac{abP_{\max}^2}{1 + aP_{\max} + bP_{\max}} \right) \end{aligned}$$

Notice that each user pays the cost because of its involvement. This cost function can be calculated and exchanged before transmission with little signalling overhead.



TABLE I: 2-user VCG  $\{U_1, U_2\}$  utility table

	$\alpha_2 = 0$	$\alpha_2 = P_{\max}$
$\alpha_1 = 0$	$\{0, 0\}$	$\{0, \log_2(1 + cP_{\max})\}$
$\alpha_1 = P_{\max}$	$\{\log_2(1 + aP_{\max}), 0\}$	$\{U'_1, U'_2\}$

### B. The Regret-Matching Algorithm

Finally, we exhibit the regret-matching algorithm [23] to learn in a distributive fashion how to achieve the correlated equilibrium set in solving the VCG auction. The algorithm is named regret-matching (no-regret) algorithm, because the stationary solution of the learning algorithm exhibits no regret and the probability to take an action is proportional to the “regrets” for not having played the other actions. Specifically, for user  $i$  there are two distinct binary actions  $\alpha_i^{(0)}$  and  $\alpha_i^{(1)}$  at every time  $t = T$  (where  $\alpha_i^{(0)} = 0$ , and  $\alpha_i^{(1)} = P_{\max}$ ). The regret  $\mathbb{R}$  of user  $i$  at time  $T$  for playing action  $\alpha_i^{(1)}$  instead of the other action  $\alpha_i^{(0)}$  is

$$\mathbb{R}_i^T(\alpha_i^{(1)}, \alpha_i^{(0)}) := \max\{D_i^T(\alpha_i^{(1)}, \alpha_i^{(0)}), 0\}, \quad (7)$$

where

$$D_i^T(\alpha_i^{(1)}, \alpha_i^{(0)}) = \frac{1}{T} \sum_{t \leq T} [U_i^t(\alpha_i^{(0)}, \alpha_{-i}) - U_i^t(\alpha_i^{(1)}, \alpha_{-i})]. \quad (8)$$

Here  $U_i^t(\alpha_i^{(\cdot)}, \alpha_{-i})$  is the utility at time  $t$  and  $\alpha_{-i}$  is other users' actions.  $D_i^T(\alpha_i^{(1)}, \alpha_i^{(0)})$  has the interpretation of average payoff that user  $i$  would have obtained if it had played action  $\alpha_i^{(0)}$  every time in the past instead of choosing  $\alpha_i^{(1)}$ . The expression  $\mathbb{R}_i^T(\alpha_i^{(1)}, \alpha_i^{(0)})$  can be viewed as a measure of the average regret. Similarly,  $\mathbb{R}_i^T(\alpha_i^{(0)}, \alpha_i^{(1)})$  represents the average regret if the alternative action has been taken.

Recalling the discussion in Subsection III-B about the mapping notation we adopted between the point mass functions  $p(\alpha_1^{(\cdot)}, \alpha_2^{(\cdot)})$  and the system time-sharing coefficients ( $\theta$ s), then we want to find the point mass function  $p(\alpha_1^{(1)}, \alpha_2^{(0)}) \equiv \theta_1$ ,  $p(\alpha_1^{(0)}, \alpha_2^{(1)}) \equiv \theta_2$ , and  $p(\alpha_1^{(1)}, \alpha_2^{(1)}) \equiv \theta_3$ . As discussed in Subsection II-D there exist  $2^n = 4$  pmfs for the 2-user case. For the trivial case of the origin point  $\Phi(0, 0)$ ,  $p(\alpha_1^{(0)}, \alpha_2^{(0)}) = 0$ . We are left to obtain  $p(\alpha_1^{(1)}, \alpha_2^{(0)})$ ,  $p(\alpha_1^{(0)}, \alpha_2^{(1)})$ , and  $p(\alpha_1^{(1)}, \alpha_2^{(1)})$ . Specifically<sup>‡</sup> to the 2-user case, this simplifies further to finding only *two* variables. Denoting  $p_1 = \theta_1$ , and  $p_2 = \theta_2$ , then  $\theta_3$  can be deduced as  $\theta_3 = 1 - p_1 - p_2$ .

The details of the regret-matching algorithm is shown in Table II. The probability  $p_i$  is a linear function of the regret, see Eq. (9). The algorithm has a complexity of  $O(|\Omega_i|) = O(2)$ .

By using the theorem in [23], if every user plays according to the learning algorithm in Table II, the adaptive learning algorithm has the property that the probability distribution found converges on the set of correlated equilibrium. It has been shown that the set of correlated equilibrium is nonempty, closed and convex [17]. Therefore, by using the algorithm in

<sup>‡</sup>Note: Solving for  $n$  variables instead of  $2^n - 1$  does not apply to  $n \geq 3$ ; the 2-user case is a special case, as  $\sum_{k=1}^{2^n-1} \theta_k = 1$  was used to simplify the unknowns to 2. For  $n \geq 3$ , see Fig.3, it is not enough to find the time-sharing strategy of *individual* users, as *ordering* needs to come into the picture in arriving to the desired *system* time-sharing coefficients.

TABLE II: Regret-matching learning algorithm

Initialize arbitrarily probability for user $i$ , $p_i$ .
For $t=2,3,4,\dots$
1. Let $\alpha_i^{t-1}$ be the action last chosen by user $i$ , and $\hat{\alpha}_i^{t-1}$ as the other action.
2. Find $D_i^{t-1}(\alpha_i^{t-1}, \hat{\alpha}_i^{t-1})$ as in Eq. (8).
3. Find average regret $\mathbb{R}_i^{t-1}(\alpha_i^{t-1}, \hat{\alpha}_i^{t-1})$ as in Eq. (7).
4. Then the probability distribution of the actions for the next period, $p_i^t$ is defined as:
$\begin{aligned} p_i^t(\hat{\alpha}_i^{t-1}) &= \frac{1}{\mu} \mathbb{R}_i^{t-1}(\alpha_i^{t-1}, \hat{\alpha}_i^{t-1}), \\ p_i^t(\alpha_i^{t-1}) &= 1 - p_i^t(\hat{\alpha}_i^{t-1}), \end{aligned} \quad (9)$
where $\mu$ is a certain constant that is sufficiently large.

Table II, we can guarantee that the algorithm converges to the set of CE as  $T \rightarrow \infty$ .

### V. SIMULATION RESULTS

To demonstrate the proposed scheme, we setup a 2-user interference channel where  $P_{\max} = 1$ . In Fig. 6, we show the crystallized rates region for the noise-limited regime with  $a = 2, b = 0.2, c = 1$ , and  $d = 0.1$ . The learning algorithm converges close to the Nash equilibrium, which means that both users transmit with maximum power  $P_{\max}$  all the time. This corresponds to the case in Fig. 2 (a). In Fig. 7, we show the Type II time-sharing case with  $a = 20, b = 2, c = 1$ , and  $d = 1$ . The algorithm converges to  $\theta_2 = 0.92$  and  $\theta_3 = 0.08$ , which means the probability that user 2 transmits alone is 0.92, and the probability that both users transmit with full power is 0.08. This corresponds to the case in Fig. 2 (c). Finally, in Fig. 8, we show the interference-limited regime with  $c = 1, d = 10$  as well as seven different instances of  $a$  and  $b$ . First, the Nash equilibriums exhibit much poorer performance than the TDMA time-sharing lines. The proposed learning algorithm converges to a point on the TDMA time-sharing lines, this corresponds to the case in Fig. 2 (d). Moreover, the learning algorithm favors the weaker user.

In Fig. 9, we show the interference-limited case with  $a = 1, b = 10, c = 1$ , and  $d = 10$ . Due to the symmetry, the learning algorithm achieves probabilities of 0.5, which means the two users conduct equal time-sharing over the channel, where each transmits solely at full power while the other is silent; and such two transmission states happen equally 50% of the time each. This corresponds to the A-C time-sharing case in Fig. 2 (d).

### VI. CONCLUSION

Treating the interference as noise, the paper proposes a novel approach to the rates region in the  $n$ -user interference channel, composed by the time-sharing convex hull of  $2^n - 1$  corner points achieved through On/Off binary power control. The resulting rates region is denoted crystallized rates region. It then applies the concept of correlated equilibrium from game theory to form the convex hull of the crystallized region. An example in applying these concepts for the 2-user case, the paper considered a mechanism design approach to design the Vickrey-Clarke-Groves auction utility function. The regret-matching algorithm is used to converge to the solution point on the correlated equilibrium set, to which subsequently simulation was presented.

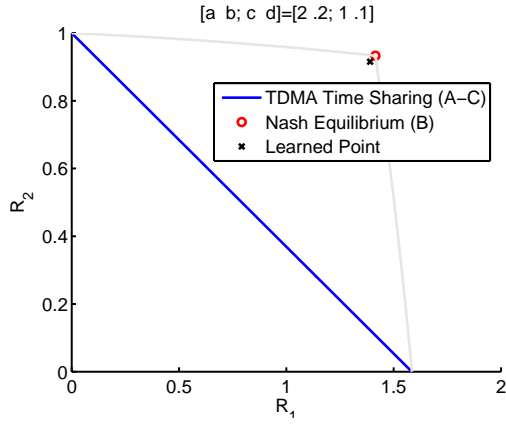


Fig. 6: Noise-limited: 2-user case

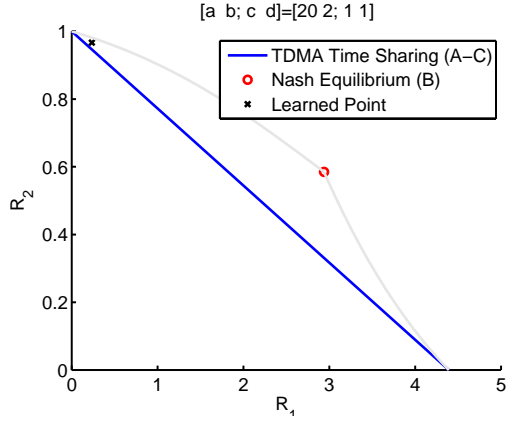


Fig. 7: Type II time-sharing (see Fig. 2 (c))

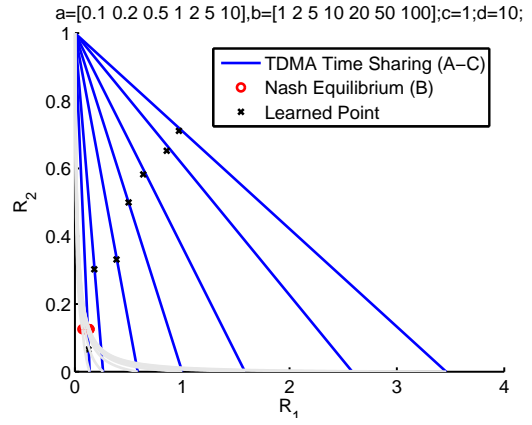


Fig. 8: Interference-limited: Type I time-sharing

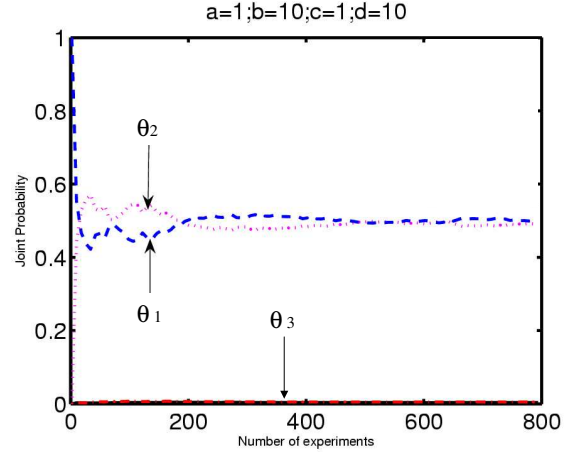


Fig. 9: Interference-limited: Type I time-sharing

## REFERENCES

- [1] R. Ahlswede, "The capacity region of a channel with two senders and two receivers", *Annals of Probability*, vol. 2, pp. 805-814, Oct. 1974.
- [2] A. B. Carleial, "Interference channels", *IEEE Transactions on Information Theory*, vol. 24, no. 1, pp. 60-70, Jan. 1978.
- [3] M. H. M. Costa, "On the Gaussian interference channel", *IEEE Transactions on Information Theory*, vol. 31, pp. 607-615, Sept. 1985.
- [4] T. Han and K. Kobayashi, "A new achievable rates region for the interference channel", *IEEE Transactions on Information Theory*, Volume 27, Issue 1, Page(s): 49 - 60, Jan 1981.
- [5] X. Shang, B. Chen and M. J. Gans, "On the achievable sum rate for MIMO interference channels", *IEEE Transactions on Information Theory*, vol. 52, no. 9, pp. 4313-4320, Sept. 2006.
- [6] R. Etkin, D. Tse and H. Wang, "Gaussian interference channel capacity to within one bit", *Arxiv preprint cs.IT/0702045*, 2007 - [arxiv.org](http://arxiv.org). [Online] <http://www.citebase.org/abstract?id=oai:arXiv.org:cs/0702045>
- [7] V. S. Annapureddy and V. V. Veeravalli, "Sum capacity of the Gaussian interference channel in the low interference regime", in *Proceedings of ITA Workshop*, San Diego, CA, Jan-Feb, 2008.
- [8] X. Shang, G. Kramer and B. Chen, "A new outer bound and noisy interference sum-rate capacity for the Gaussian interference channels", Submitted to *IEEE Transactions on Information Theory*, Dec. 2007.
- [9] A. S. Motahari and A. K. Khandani, "Capacity bounds for the Gaussian interference channel", submitted to *IEEE Transactions on Information Theory*.
- [10] M. Charafeddine, A. Sezgin, and A. Paulraj, "Rates region frontiers for  $n$ -user interference channel with interference as noise", in *Proc. of Annual Allerton Conference on Communication, Control, and Computing*, Allerton, IL, Sep. 2007. [Online] <http://www.csl.uiuc.edu/allerton/archives/allerton07/PDFs/papers/0048.pdf>
- [11] E. G. Larsson and E. A. Jorswieck, "Competition versus cooperation on the MISO interference channel", *IEEE Journal on Selected Areas on Communications*, vol. 26, pp. 1059-1069, Sept. 2008.
- [12] E. Jorswieck and E. Larsson, "The MISO interference channel from a game-theoretic perspective: A combination of selfishness and altruism achieves Pareto optimality", in *proc. of ICASSP 2008*, invited.
- [13] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Uniform power allocation in MIMO channels: A game-theoretic approach", *IEEE Transactions on Information Theory*, vol. 49, no. 7, p.p. 1707-1727, Jul. 2003.
- [14] Z. Han, Z. Ji, and K. J. R. Liu, "Non-cooperative resource competition game by virtual referee in multi-cell OFDMA networks", *IEEE Journal on Selected Areas in Communications*, Special Issue on Non-cooperative Behavior in Networking, vol.53, no.10, p.p.1079-1090, Aug. 2007.
- [15] W. Yu, G. Ginis, and J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines", *IEEE Journal on Selected Areas in Communications*, vol.20, no.5, pp.1105-1114, Jun. 2002.
- [16] G. Owen, *Game Theory*, 3rd ed. New York: Academic, 2001.
- [17] R. J. Aumann, "Subjectivity and correlation in randomized strategy", *Journal of Mathematical Economics*, vol. 1, no. 1, pp. 67-96, 1974.
- [18] E. Altman, N. Bonneau and M. Debbah, "Correlated equilibrium in access control for wireless communications", In *Proc. of Networking*, Coimbra, Portugal, May 2006.
- [19] E. Altman, K. Avrachenkov, N. Bonneau, M. Debbah, R. El-Azouzi and D. S. Menasche, "Constrained stochastic games in wireless networks", in *Proc. of IEEE Globecom*, Washington D.C., US, Nov., 2007.
- [20] Z. Han, C. Pandana, and K. J. R. Liu, "Distributive opportunistic spectrum access for cognitive radio using correlated equilibrium and no-regret learning", in *Proceedings of IEEE Wireless Communications and Networking Conference*, Hong Kong, China, March 2007.
- [21] V. Krishna, *Auction Theory*, Academic Press, 2002.
- [22] C. Papadimitriou, "Computing correlated equilibria in multiplayer games", [Online] <http://www.cs.berkeley.edu/~christos/>.
- [23] S. Hart and A. Mas-Colell, "A simple adaptive procedure leading to correlated equilibrium", *Econometrica*, vol. 68, no. 5, pp. 1127-1150, Sep. 2000.
- [24] Z. Han and K. J. R. Liu, *Resource Allocation for Wireless Networks: Basics, Techniques, and Applications*, Cambridge University Press, 2008.